Fuzzy De-noise of ECG signals with wavelet techniques

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Abstract. This work describes an adaptive fuzzy scheme for noise suppression of electrocardiograms (ECG). The Wavelet Transform (WT) is used for analysis and synthesis of signals in the noise cancellation process. A Takagi-Sugeno-Kang system carries out the diffuse adjustment of the threshold operator involved in the suppression of noise.

Key words: Fuzzy de-noise, threshold, TSK, electrocardiograms, wavelets.

1. Introduction

Electrocardiogram (ECG) is a graphical register of the electrical potentials produced cardiac tissue, which ones are located between a frequency range from 1 to 100 Hz. the synchronization mechanism is shown on Figure 1.

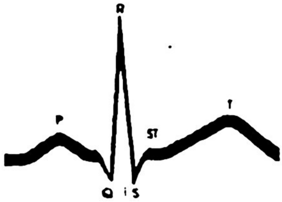

Fig. 1 Electrocardiogram elemental signal.

The bents presents in the drawing of an ECG represent different states of the heart ing a beat. The *P wave* and the *QRS complex* indicates contraction of the atriums ventricles respectively, the *ST segment* indicate the time that transcurse since a ventri contraction ends to a starting rest period previous to the ventricles start to contract selves for the next beat, and finally the *T wave* indicate the rest period of the ventri [5].

The micro-electrical signal analysis has been strongly used to determine different kinds of medical malaises and diseases such as muscular fatigue, cardiac dysfunctions, etc. Y thanks to the advance of this area, actually it is possible to realize medical diagnostics.

A motivation behind the implementation of transforms in signal processing is that for some applications signals in this domain offers better information in analysis that in the time domain [13].

Applications of the wavelet transform in signal processing in biomedical engineering include a great variety of tasks. In cardiology, analysis with wavelets is applied in electrocardiograms [1, 3, 7, 18].

Utilization of Fuzzy Logic provides machinery for carrying out approximate reasoning processes when available information is uncertain, incomplete, imprecise, or vague. The success of this methodology has been demonstrated in a variety of fields, such as control of systems, experts systems [17].

The aim in this work is to achieve a fuzzy de-noise in signals coming from electrocardiograms; implementing the WT and a threshold operator, with the purpose to obtain a signal free of noise.

The work is organized in the following way. Section 2 presents the scheme used to achieve the de-noise of the ECG signal, section 3 shows the methodology used in the elaboration of the work, section 4 shows the results obtained in simulation, and finally on section 5 are located the accomplished conclusions from results shown during work development.

Notation

This preliminary section fixes the mathematic notation used in the paper.

t	Continues time variable,	$t \in R$
n	Discrete time variable,	$n \in Z$
χ	Hilber pace,	
$\langle , \rangle$	Inner product, $\langle , \rangle : \chi, \chi \to C$	
l_2	$l_2 = \left\{ X[n] \mid \sum_{n=-\infty}^{\infty} \mid X[n] \mid^2 \right\} < \infty$	
L_2	$L_2 = \left\{ f[t] \mid \int_{-\infty}^{\infty} f(t) \mid^2 dt \right\} < \infty$	

2. Fuzzy - Wavelet scheme

In this section the fuzzy – wavelet scheme used for noise suppression is shown. Firstly summary of basic notions on wavelet transform, and later the fuzzy model used is presented.

2.1 Wavelet Transform (WT)

The continuous wavelet transform of a function *f(t)* in a Hilbert space X , is defined follows:

$$CWT_f(a,b) = \langle \psi_{a,b}(t), f(t) \rangle, f(t) \in L_2(R) \quad (1$$

Where

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \quad (2)$$

Is the basic function called mother wavelet, which one depends of the parameters a, where both are used to transfer and to climb respectively.
For the discrete case the wavelet transform for a discrete signal $x[n]$ is given by

$$DWT_f(a,b) = \langle \psi_{a,b}[n], x[n] \rangle = \sum_{-\infty}^{\infty} x[n] \psi_{a,b}[n], x[n] \in l_2(R) \quad (3$$

with sampled time, T and $n = kT$.

A general model for noise suppression using wavelets is depicted in Figure 2[14].

Fig. 2. Model for noise suppression.

In Figure 2, the first processing applied to the signal consists in an analysis based wavelets; with this analysis the signal is decomposed in Low and High frequencies, threshold operator acts in high frequencies and in this point noise suppression is formed. The threshold operator is fixed by a fuzzy system; finally a wavelet synthesis carried out for to yield a signal free of noise.

The analysis and synthesis process are shown in Figure 3; in the which one can obse that the signal is decomposed in high and low frequencies by one couple of filters wavelet coefficients, after it a downsampling and the threshold operator are applied. nally a subsampling and the synthesis process are performed with the dual filters $\overline{H}$ [14].

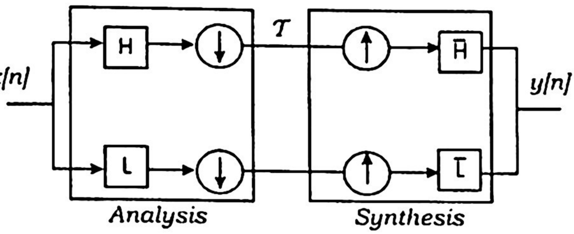

Fig. 3. Analysis and Synthesis in the wavelet scheme.

2.2 Fuzzy Model

The fuzzy model proposed by Takagi-Sugeno-Kang is described by fuzzy rules IF() and IF() .. THEN, each one is a lineal relationship between inputs and outputs. This set of rules is expressed under the form:

$$R_j = IF\ x\ is\ A_j \quad THEN \quad y = f_j(x)$$

Here A_j are linguistic terms, x is the input linguistic variable, while $y = (y_1, \ldots, y_j, \ldots y_{\max})$ is the output variable. The value of the input linguistic variable may be crisp or fuzzy. If the value of the input variable is a crisp number then the variable x is called a singleton. The TSK system is used to achieve adjusts of the threshold operator dynamically, over each decomposition in the wavelet scheme.

3. Methodology

In this section two kinds of signals were used; reference signal and noisy signal. The reference signal is ECG signal without noise, the noisy signal is filtered with the fuzzy – wavelet scheme to approximate it to the reference signal.

The carried out process is illustrated in the following steps:

- Discrete Wavelet Transform (DWT) compute.
- Determination of the noise (entropy, energy).
- Filtering process with threshold operator.
- Inverse Discrete Wavelet Transform (IDWT) compute.
- Comparison between the reference letter and the IDWT, with the purpose of calculating the reconstruction quality.

The DWT was computed with the analysis wavelet presented in Figure 3, the ECG signal was decomposed in low frequencies (approaches) and high frequencies (details), after a subsampling process was applied, and as result, a coarse version of the original signal was obtained.By empirical approaches was determinate two decomposition levels for the analysis process and the wavelet of analysis was the wavelet of Daubechies 10.

In order to determinate the levels of noise presented in the ECG signal, two parameters were considered entropy and energy. Entropy and energy were calculated from details (high frequencies) in the signal. High entropy indicates noise in the signal and if this energy is not too large, noise has a relatively small influence on the important large signal coefficients. These observations suggest that small coefficients should be replaced by zero, because they are dominated by noise and carry only a small amount of information. Both parameters are used to adjust the threshold operator.

Noise suppression is accomplished with the threshold operator shown in Figure 2, this operator attenuates or keeps components in signal.

The threshold operator is fixed with the TSK system, which depends of energy and entropy in the signal; both parameters are represented by five membership functions shown in Figure 4. Besides twenty five (25) IF ... THEN rules are generated. The minimum is used as *t-norm* in the reasoning process, to determinate the best threshold.

The set of fuzzy rules is shown in Table 1, the rules has been built of smaller to bigger, below to up, and left to right. In this way, **IF** the *energy* is *smaller* and the *entropy* is *smaller* **THEN** the threshold operator should be pondered by 0.01.

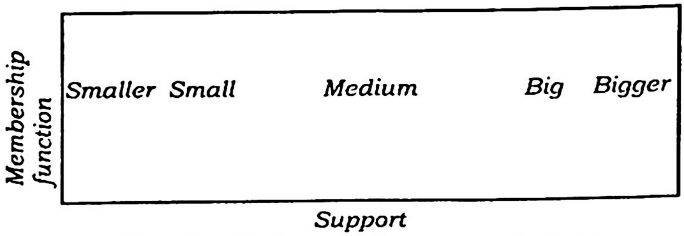

Fig. 4 Membership functions for energy and entropy.

In order to apply this method, it is necessary to consider that the used signals are long play, like for example the Holter signals. In this case, the methods applied can fail because throughout the signal the noise level will change. A possible solution is to divide the signal in a series of intervals, and to process each one of them separately.

Table 1. Set of fuzzy rules.

0.10	0.15	0.30	0.45	0.50
0.15	0.30	0.45	0.50	0.55
0.30	0.45	0.50	0.55	0.70
0.45	0.50	0.55	0.70	0.85
0.50	0.55	0.70	0.85	1.00

3.1 Threshold Operator

Thresholding is performed so as to zero out small magnitude wavelet coefficients and retain the value of large magnitude coefficients. The general soft threshold operator is defined as [14]:

$$\left(F_{\delta}c\right)_n = \begin{cases} c[n], & |c[n]| > \delta, \\ 0, & otherwise \end{cases} \tag{4}$$

Where $c[n]$ are the details (high frequencies). The threshold δ proposed is given by

$$\delta = \sqrt{2\log(N)\bar{\sigma}} \tag{5}$$

where N is the level of decomposition and $\bar{\sigma}$ estimation of the noise variance

$$\bar{\sigma} = median\left(|c[n]|\right)/0.6745 \tag{6}$$

The calculation of inverse discrete transformed wavelet (IDWT) for the reconstruction of the signal is made firstly by means of the process of synthesis shown in Figure 3, consistent in an upsampling, followed of a convolution with the filters wavelet L and H.

Finally a comparison between the reference signal and the filtered signal is made to determine the effectiveness of the filtrate. This comparison is made taking the measurement from the mean square error:

$$E(e) = \frac{1}{N}\sum_{n=0}^{N}(e)^2 \tag{7}$$

Where e is the difference between the reference signal and the filtered signal, i.e. $e[n] = d[n] - y[n]$

Table 2. De-noise schemes.

De-noise Scheme	Error
Soft threshold	0.02835
Fuzzy threshold	0.02640

4 Lab's results

For the implementation of these schemes were used a set of ECG signals that previously were obtained from the PhysioNet's network database[1]. Comparison between the soft threshold and the fuzzy threshold was made, and the results are shown on Table 2. The fuzzy threshold has relatively a better performance, with the reference signal.

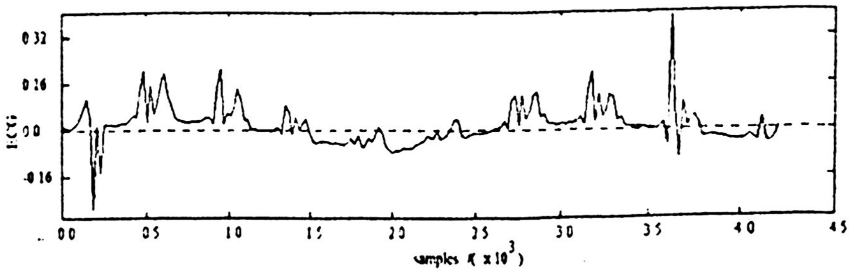

Fig. 5 Filtered signal with soft threshold.

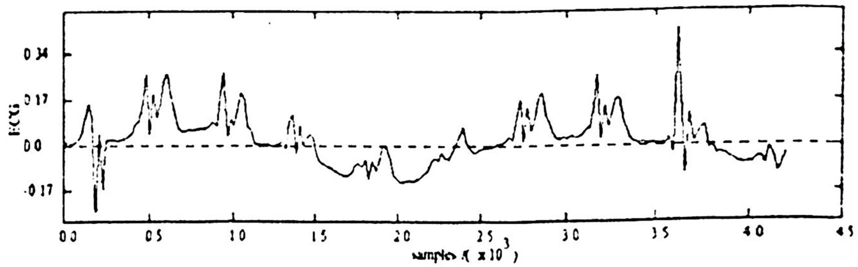

Fig. 6. Filtered signal with fuzzy threshold.

[1] www.physionet.org

Signal obtained by both schemes are shown in Figures 5 and 6. Figure 5 represent the filtered signal with the soft threshold, and Figure 7 represents the filtered signal with fuzzy threshold.

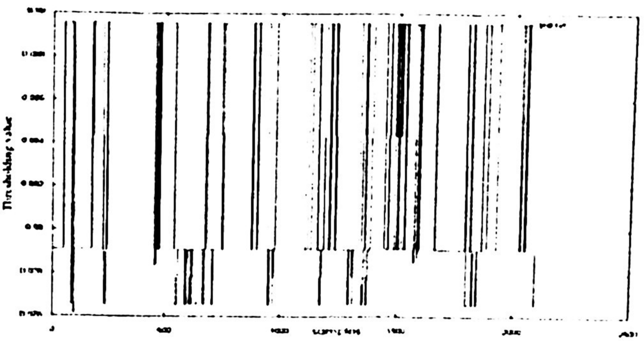

Fig. 7 Thresholding level 1.

The behavior presented by the threshold operator in the first level of decomposition appears in Figure 7, where it is possible to observe that its value is changing according to the values of entropy and energy during time windows, these windows of time are of 11 samples. Besides is possible to observe that Figure 8 presents a time scale equal to half of the original signal, because the threshold operator acts on the coefficients of high frequency in each level of decomposition where every time the amount of samples is reduced to half.

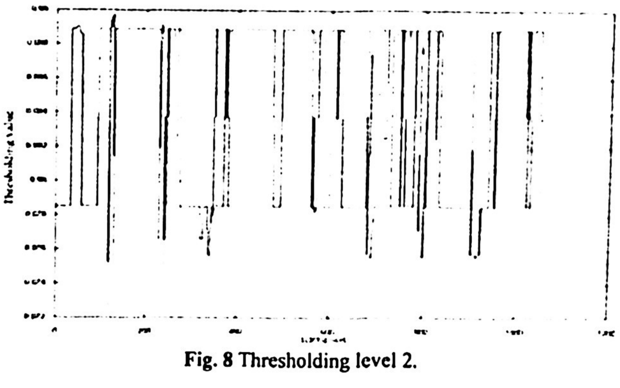

Fig. 8 Thresholding level 2.

Finally in Figure 9 the performance of the threshold operator for the second level of decomposition is shown, and in the same way its value is varying during the time.

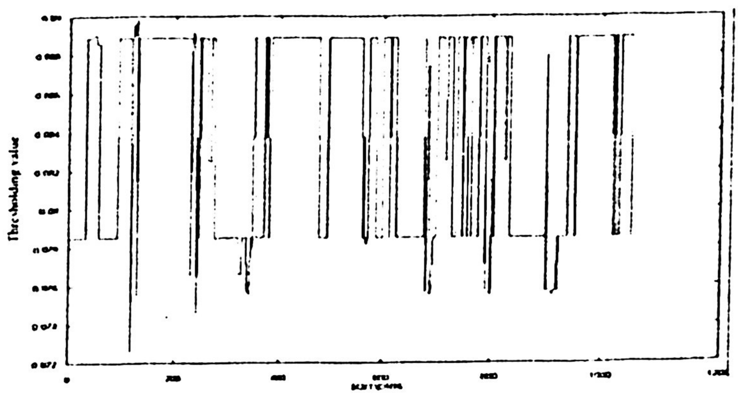

Fig. 9 Thresholding level 2.

5 Conclusions and Outlooks

In this work is proposed a scheme and algorithm of adapted filtering. With the properties of the wavelet transform, was possible to implement a combined digital filter with the threshold operator. In the other hand the implementation of a fuzzy system TSK was made, to adjust dynamically the threshold operator, which took in consideration physical properties from the signal, such as the entropy and the energy.

The performance of the scheme presented, show better results with respect to the classic scheme of noise suppression with wavelets (soft threshold)[3]. This performance can be improved, adjusting the set of fuzzy rules of TSK system, with more properties of ECG signals. In addition linguistic attributes of the signals can be used.

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